2019

MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×10=10

   (a) Write down the degree of the differential equation

   $$\frac{dy}{dx}+x=\left(y-x\frac{dy}{dx}\right)^{-3}$$

   (b) What do you mean by singular solution of a differential equation?

   (c) Is the integrating factor of the differential equation $x\,dy-y\,dx=0$ unique?

(d) If Q is zero, write the general solution of the first-order linear differential equation

$$\frac{dy}{dx} + Py = Q$$

where P and Q are the functions of x alone or constant.

(e) Which of the following functions is the solution of the differential equation $\frac{d^2y}{dx^2} = 0$?

(i) $y = 5x$

(ii) $y = 5x + 6$

(iii) $y = e^{5x}$

(iv) $y = e^{5x} + 6$

(Choose the correct option)

(f) Write down the particular integral of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^{-x}$$

(g) Write down the general solution of the differential equation $y = px + \frac{2}{p}$, where $p = \frac{dy}{dx}$.

(h) Write the differential equation of orthogonal trajectories for a family of curves given by the differential equation

$$f\left(x, y, \frac{dy}{dx}\right) = 0$$

(i) Write the standard form of the linear partial differential equation of order one.

(j) Write the conditions for exactness of the total differential equation

$$P(x, y, z)\,dx + Q(x, y, z)\,dy + R(x, y, z)\,dz = 0$$

2. Answer the following questions : 2×5=10

(a) Solve :

$$(x^2 - y^2)\,dx + 2xy\,dy = 0$$

(b) Find the equation of the curve represented by

$$(y - yx)\,dx + (x + xy)\,dy = 0$$

and passing through the point (1, 1).

(c) Solve :

$$\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = 0$$

(d) Construct a partial differential equation by eliminating a and b from

$$z = (x^2 + a)(y^2 + b)$$

where z be a function of two independent variables of x and y.

(e) If $8(x + ay + b)^3 = (1 + a^3) z^2$ is the complete integral of a partial differential equation $(p^3 + q^3) = 27z$, find its singular integral.

3. Answer any *four* parts : 5×4=20

(a) Prove that necessary and sufficient condition that a differential equation $M dx + N dy = 0$ be exact is that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

(b) Solve

$$yp^2 + (x-1)p - x = 0$$

where $p = \frac{dy}{dx}$.

(c) Find the orthogonal trajectories of the family of curves

$$(x^{2/3} + y^{2/3}) = a^{2/3}$$

where a is parameter.

(d) Solve :

$$x^2 \frac{d^2y}{dx^2} - (x^2 + 2x)\frac{dy}{dx} + (x+2)y = x^3 e^x$$

(e) Solve :

$$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = x$$

Given that $y = 0$ when $x = 1$ and $y = e^2$ when $x = e$.

(f) Solve :

$$xz^3 dx - z dy + 2y dz = 0$$

4. Answer *either (a)* and *(b) or (c)* and *(d)* : 5+5=10

(a) Obtain the equation of the curve whose slope at any point is equal to $y + 2x$ and which passes through the origin.

(b) Solve the following differential equation by reducing it in linear form

$$(x - y^2) dx + 2xy dy = 0$$

(c) Reduce the differential equation

$$y = 2px + yp^2$$

where $p = \frac{dy}{dx}$, to Clairaut's form by substituting $y^2 = v$ and hence solve the equation.

(d) Solve :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0$$

Given that $y(0) = 4$, $\frac{dy}{dx} = 1$ at $x = 0$.

5. Answer *either* (a) and (b) *or* (c) and (d) : 5+5=10

(a) Reduce the differential equation

$$\frac{d^2y}{dx^2} - 2\tan x\frac{dy}{dx} + 5y = \sec x . e^x$$

to its normal form and hence solve it.

(b) Solve :

$$u\,dx = (u - 2x)\,du$$
$$u\,dy = (ux + uy + 2x - u)\,du$$

(c) Solve :

$$\sin^2 x\frac{d^2y}{dx^2} = 2y$$

Given that $y = \cot x$ is a solution.

(d) Find the necessary condition for integrability of the total differential equation

$$P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$$

6. Answer *either* (a) and (b) *or* (c) and (d) : 5+5=10

(a) Apply variation of parameters to solve the differential equation

$$\frac{d^2y}{dx^2} + y = x$$

(b) Solve :

$$\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$$

(c) Solve

$$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + \frac{a^2}{x^4}y = 0$$

by changing the independent variable x to z.

(d) Form a partial differential equation by eliminating the arbitrary function ϕ from

$$\phi(x + y + z, x^2 + y^2 - z^2) = 0$$

7. Answer *either* (a) and (b) *or* (c) and (d) : 5+5=10

(a) Solve by Lagrange's method

$$p + q = x + y + z$$

where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

(b) Find the complete integral of

$$z^2(p^2z^2+q^2)=1$$

where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$.

(c) Solve by Charpit's method

$$z=px+qy+pq$$

where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$.

(d) Find the integral surface of the partial differential equation

$$(x^2-yz)p+(y^2-zx)q=(z^2-xy)$$

which passes through the line $x=1$, $y=0$, where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$.

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